JEE Advanced 2018 Paper-1

These 17 Questions out of 18 are taught directly in classroom by Prof. Mukul Jha

Q.1 The potential energy of a particle of mass m at a distance r from a fixed point O is given by $V(r) = kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true?

(A)
$$v = \sqrt{\frac{k}{2m}} R$$
 (B) $v = \sqrt{\frac{k}{m}} R$ (C) $L = \sqrt{mk} R^2$ (D) $L = \sqrt{\frac{mk}{2}} R^2$

Sol: Same As Taught Directly in Circular Motion with conservative Force

:.Ans(B,C)

Force, $F = - dv/dr = - d(kr^2/2)/dr = -k .2r/2 = -kr$

:. F = 2kr, attractive (-ve sign)

Newton's Law,
$$\Sigma$$
 $\mathbf{F}_{\text{ext}} = \text{ma} => \text{kr} = \text{mv}^2/\text{r}$:.v = r $\sqrt{\text{k/m}}$:. Ans(B) correct L = mvr = mr $^2\sqrt{\text{k/m}}$ = r $^2\sqrt{\text{mk}}$:. Ans(C) correct

- **Q.2** Consider a body of mass 1.0 kg at rest at the origin at time t = 0. A force $\mathbf{F} = (\alpha t \mathbf{i} + \beta \mathbf{j})$ is applied on the body, where $\alpha = 1.0$ N/s and $\beta = 1.0$ N. The torque acting on the body about the origin at time t = 1.0 s is τ . Which of the following statements is (are) true?
- (A) $|\tau| = 1/3 \text{ N m}$
- (B) The torque τ is in the direction of the unit vector + k
- (C) The velocity of the body at t = 1 s is $\vec{v} = 1/2$ (i + 2i) m/s
- (D) The magnitude of displacement of the body at t = 1 s is 1/6 m

Sol: Same As Taught Directly in Theory Class of Rotational Dynamics

:.Ans(A,C)

Q.3 A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ . The angle of contact between water and the wall of the capillary tube is θ . Ignore the mass of water in the meniscus. Which of the following statements is (are) true?

- (A) For a given material of the capillary tube, h decreases with increase in r
- (B) For a given material of the capillary tube, h is independent of σ
- (C) If this experiment is performed in a lift going up with a constant acceleration, then h decreases
- (D) h is proportional to contact angle θ

Sol: Same As Taught Directly in class of Simple Tension

:.Ans (A, C)

$$h = 2\sigma \cos\theta / \rho gr$$
 h

h α 1/r

Ans (A)

h α $cos\theta$ Ans (D) wrong

Also h depends on σ Ans (B) is wrong

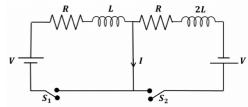
If accelerated up in lift, g will be increased (g+a)

 $h' = 2\sigma \cos\theta / \rho (g + a) r$, h will decrease Ans (C)

0.4 In the figure below, the switches S1 and S2 are closed simultaneously at t = 0 and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its

maximum magnitude I_{max} at time $t = \tau$. Which of the following statements is (are) true?

- (A) $I_{max} = V/2R$
- (B) $I_{max} = V/4R$
- (C) $\tau = L/R \ln 2$
- (D) $\tau = 2L/R \ln 2$



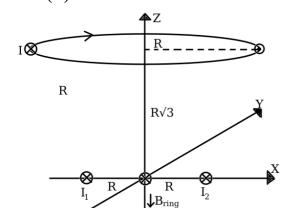
Sol: Ans:(B,D)

- Q.5 Two infinitely long straight wires lie in the xy-plane along the lines $x = \pm R$. The wire located at x = +R carries a constant current I_1 and the wire located at x = -R carries a constant current I_2 . A circular loop of radius R is suspended with its centre at $(0, 0, \sqrt{3}R)$ and in a plane parallel to the xyplane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive if it is in the $+\mathbf{i}$ direction. Which of the following statements regarding the magnetic field **B** is (are) true?
- (A) If $I_1 = I_2$, then **B** cannot be equal to zero at the origin (0, 0, 0)
- (B) If $I_1 > 0$ and $I_2 < 0$, then **B** can be equal to zero at the origin (0, 0, 0)
- (C) If $I_1 < 0$ and $I_2 > 0$, then **B** can be equal to zero at the origin (0, 0, 0)
- (D) If $I_1 = I_2$, then the z-component of the magnetic field at the centre of the loop is (- μ_0 I/ 2R)

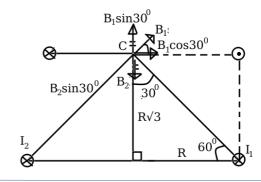
Sol: Same As Taught Directly as Q[10] HCV in Electromagnetism

:.Ans(A,B,D)

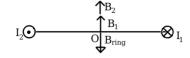
(A) If $I_1 = I_2$, both (x) or (.) B_1 will cancel B_2 But Bring will remain :. B not equal to 0 :.Ans(A) is correct

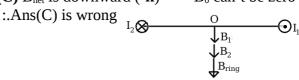


- **(D)** At C, $B_1 \sin 30^\circ$ will cancel $B_2 \sin 30^\circ$ but $B_1 \cos 30^{\circ} + B_2 \cos 30^{\circ}$
- $=2B_1 \cos 30^\circ = B_1\sqrt{3}$ will remain as B_X $B_z = -\mu_0 I/ 2R$, due to ring only :.Ans(D) is correct

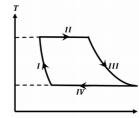


(B) B_1 and B_2 upward can cancel B_{ring} will remain **(C)** B_{net} is downward (-**k**) -> B_0 can't be zero :. Ans (B) is correct





- Q.6 One mole of a monatomic ideal gas undergoes a cyclic process as shown in the figure (where V is the volume and T is the temperature). Which of the statements below is (are) true?
- (A) Process I is an isochoric process
- (B) In process II, gas absorbs heat
- (C) In process IV, gas releases heat
- (D) Processes I and III are not isobaric



Sol: Same As Taught Directly in Theory class of Thermodynamic Processes

:.Ans(B,C,D)

Process (I) is not isochoric (A) wrong.

In (IV) -> T is costant

pV = nRT = constant

Also, Isothermal, $\Delta V=0$

 $\Delta Q = \Delta W = \text{-ve (compression)}$

:. Heat is released Ans(C)is correct

In (II) -> Isothermal, ΔU =0 $\Delta Q = \Delta W = +ve$, heat is absorbed (B) is correct

For isobaric -> p constant $V \alpha T =>$ Linear which is not in graph.

Q.7 Two vectors **A** and **B** are defined as **A** =a **i** and **B** = a (cos ωt **i** + sin ωt **j**), where a is a constant and $\omega = \pi/6$ rad/s . If $|\mathbf{A} + \mathbf{B}'| = \sqrt{3}|\mathbf{A} - \mathbf{B}'|$ at time $t = \tau$ for the first time, the value of τ , in seconds, is

Sol: Same As Taught Directly in Vectors and SHM

Ans: 2 sec

$$|\mathbf{A} + \mathbf{B'}| = \sqrt{3}|\mathbf{A} - \mathbf{B'}| = \sqrt{[(a + a \cos \omega t)^2 + (a \sin \omega t)^2]} = \sqrt{3}\sqrt{[(a - a \cos \omega t)^2 + (a \sin \omega t)^2]}$$

$$\sqrt{(1 + 1 + 2 \cos \omega t)} = \sqrt{3}\sqrt{(1 + 1 - 2 \cos \omega t)} = \sqrt{2}\sqrt{[2 \cos^2(\omega t/2)]} = \sqrt{3}.\sqrt{2}\sqrt{2}\sin^2(\omega t/2)$$

$$\cos(\omega t/2) = \sqrt{3}.\sin(\omega t/2) = \tan(\omega t/2) = 1/\sqrt{3} = \cos(\omega t/2) = \pi/6$$

$$= \frac{\pi}{6}. t/2 = \frac{\pi}{6} = t = 2 \sec$$

Q.8 Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed 1.0 m/s and the man behind walks at a speed 2.0 m/s . A third man is standing at a height 12 m above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air is 330 m/s . At the instant, when the moving men are 10 ? apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is

Sol: Same As Taught Directly in classroom of Doppler's Effect in Sound

Ans: 5 Hz

Doppler's Effect in sound,

$$v_1 = (V - 0) / (V - 2\cos\theta) \times 1430$$

= $(330 \times 1430)/(330 - 2(5/13))$
= $[(330 \times 1430 \times 13) / 4280] \text{ Hz}$
 $v_2 = (V - 0) / (V + 1\cos\theta) \times 1430$
= $(330 \times 1430)/(530 + 5/13)$
= $[(330 \times 1430 \times 13) / 4295] \text{ Hz}$
:. Beat frequency = $|v_1 - v_2| = 5.00 \text{ Hz}$

13m 0sθ 12m

Q.9 A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2 - \sqrt{3}) / \sqrt{10}$ s, then the height of the top of the inclined plane, in meters , is ______. Take g = 10 m/s².

Sol: Same As Taught Directly in theory class of Pure Rolling Motion

$$a = g \sin\theta / (1 + k^2/r^2)$$

For ring,
$$a_1 = g \sin\theta / (1 + 1) = g \sin\theta / 2$$

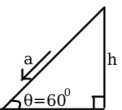
For disc,
$$a_2 = g \sin\theta / (1 + 1/2) = 2g \sin\theta / 3$$

$$S = ut + 1/2 \cdot at^2 = 1/2 \cdot at^2$$

$$\underline{\text{ring}} - h / \sin \theta = 1/2$$
. (g $\sin \theta / 2$) $t_1^2 :. t_1 = \sqrt{4h/g} \sin^2 \theta = \sqrt{16h/3g}$

$$\underline{\text{disc}} \to h / \sin \theta = 1/2$$
. (2g $\sin \theta / 3$) $t_2^2 :: t_1 = \sqrt{3}h/g \sin^2 \theta = \sqrt{4}h/g$

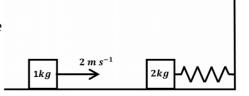
Ans: <u>0.75 m</u>



Now,
$$t_1 - t_2 = (2 - \sqrt{3}) / \sqrt{10}$$
 sec

$$\sqrt{(h/g)}$$
. $\{4/\sqrt{3}-2\}=(2-\sqrt{3})/\sqrt{10}$ taking $g=10$, solving for h we get, $h=3/4=0.75$ m

O.10 A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 N/m and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 m/s collides elastically with the first block. The collision is such that the

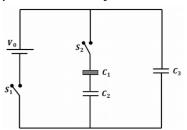


2.0 kg block does not hit the wall. The distance, in meters, between the two blocks when the spring returns to its unstretched position for the first time after the collision is _

Sol: Same As Taught Directly in class of Q. 42 of centre of Mass Chapter of HCV

For elastic collision, $v_1 = [(1-2)/(1+2)] \times 2 = -2/3 \text{ m/s}$, $v_2 = [(2x1)/(1+2)] \times 2 = 4/3 \text{ m/s}$ When 2 kg reaches mean position back, $t = T/2 = 1/2 \cdot 2\pi \cdot \sqrt{(m/k)} = \pi \cdot \sqrt{2/2} = \pi$ sec Distance = v_1 . $\pi = 2/3 \pi = 2 \times 3.14 / 3 = 6.28 / 3 = 2.09 m$

Q.11 Three identical capacitors C_1 , C_2 and C_3 have a capacitance of 1.0 μ F each and they are uncharged initially. They are connected in a circuit as shown in the figure and C₁ is then filled completely with a dielectric material of relative permittivity ϵ_r . The cell electromotive force (emf) $V_0 = 8 \text{ V}$. First the switch C_1 is closed while the switch C_2 is kept open. When the capacitor C₃ is fully charged, C₁ is opened and C₂ is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5 μ C. The value of $C_r =$

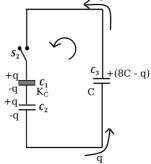


Sol: Same As Taught Directly in class as Question of Irodov Class Set (3) S_1 is closed, C_1 will be charged $\Rightarrow Q_1 = C_1 \cdot E = C \cdot 8$

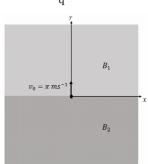
Now when S_2 is closed, by loop

$$\frac{8C - q}{C} - \frac{q}{kC} - \frac{q}{C} = 0$$
 => 5 _ 3C _ 3C _ 3C = 0 => k = 3/2 = 1.5

Also
$$(8C - q)/C = 5 \text{ volt} => 8C - q = 5C => q = 3C$$
Ans: 1.50



Q.12 In the xy-plane, the region y > 0 has a uniform magnetic field B_1 **k** and the region v < 0 has another uniform magnetic field B_2 **k**. A positively charged particle is projected from the origin along the positive ?-axis with speed $v_0 = \pi$ m/s at t = 0, as shown in the figure. Neglect gravity in this problem. Let t = T be the time when the particle crosses the ?-axis from below for the first time. If B_1 = 4B₁, the average speed of the particle, in m/s, along the x-axis in the time interval T is _



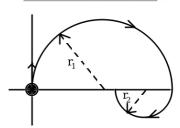
Sol: Same As Taught Directly as in Theory class of Cyclotron Ans: 1.50 Average feed along x - axis,

$$V_X = \sum x / \sum t = (2r_1 + 2r_2) / (t_1 + t_2), \text{ where } v = \pi \text{ m/s}$$

$$r_1 = mv / qB_1, r_2 = mv / qB_2 = mv / 4qB_1 \text{ as } B_2 = 4B_1$$
 Time to cover B_1 zone, $t_1 = T_1/2 = \pi m / qB_1$

Time to cover
$$B_2$$
 zone, $t_2 = T_2/2 = \pi m / qB_2 = \pi m / 4qB_1$

$$V_X = \frac{2 \text{ mv} / qB_1 + 2 \text{ mv} / 4qB_1}{\pi \text{m} / qB_1 + \pi \text{m} / 4qB_1} = \frac{(8+2) \text{ v}}{(4+1) \pi} = 2.00 \text{ m/s}$$



Q.13 Sunlight of intensity 1.3 kW/m² is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kW/m², at a distance 22 cm from the lens on the other side is

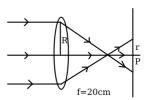
Sol: Same As Taught Directly in class in Intensity through lenses

Comparing the total energy per sec.

Energy icident per second = I . A = 1.3 x π R² (kW)

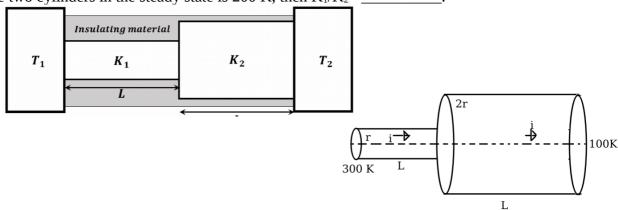
This much energy per sec will be incident over the area πR^2 at 22 cm. :. Intensity observed at P, I' = 1.3 $\pi R^2 / \pi r^2 \text{ kW/m}^2 = 1.3 \text{ R}^2 / r^2 \text{ kW/m}^2$

=
$$1.3 \text{ x}(20/2)^2$$
, by similar Δ s
= $1.3 \text{ x} 100 = 130 \text{ kW/m}^2$



Ans: 130 kW/m²

Q.14 Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures $T_1 = 300$ K and $T_2 = 100$ K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady state is 200 K, then $K_1/K_2 =$ ______.



Sol: Same As Taught Directly in Heat Transfer as Series Combination of Thermal Conductors.

Ans: <u>4</u>

In series Thermal current i is same in both conductors i =
$$\{K_1 . \pi r^2 (300-200)\} / L = \{K_2 . \pi (2r)^2 (200-100)\} / L => K_1 = 4 K_2 :. K_1 / K_2 = 4.00$$

PARAGRAPH - "X"

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, [E] and [B] stand for dimensions of electric and magnetic fields respectively, while $[\epsilon_0]$ and $[\mu_0]$ stand for dimensions of the permittivity and permeability of free space respectively. [L] and [T] are dimensions of length and time respectively. All the quantities are given in SI units

Q.15 The relation between [E] and [B] is

(A)
$$[E] = [B] [L] [T]$$
 (B) $[E] = [B] [L]^{-1} [T]$ (C) $[E] = [B] [L] [T]^{-1}$ (D) $[E] = [B] [L]^{-1} [T]^{-1}$

Sol: Same As Taught Directly in Electromagnetic Waves

$$E/B = C : [E] = [B . C] = [B] [L] [T]^{-1}$$

Ans: (C)

Q.16 The relation between $[\varepsilon_0]$ and $[\mu_0]$ is

(A)
$$[\mu_0] = [\epsilon_0] [L]^2 [T]^{-2}$$

(C)
$$[\mu_0] = [\epsilon_0]^{-1} [L]^2 [T]^{-2}$$

(B)
$$[\mu_0] = [\epsilon_0] [L]^{-2} [T]^{-2}$$

(D)
$$[\mu_0] = [\epsilon_0]^{-1} [L]^{-2} [T]^2$$

Sol: Same As Taught Directly in Electromagnetic Waves

Ans: (D)

Speed of light in vaccum,
$$c = 1 / \sqrt{(\mu_0 \, \epsilon_0)} = [LT^{-1}]^2$$
 :. $[\mu_0] = [\epsilon_0]^{-1} \, [L]^{-2} \, [T]^2$

$$[\mu_0] = [\epsilon_0]^{-1} [L]^{-2} [T]^2$$

PARAGRAPH "A"

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation z = x/y. If the errors in x, y and z are Δx , Δy and Δz , respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y} \left(1 \pm \frac{\Delta x}{x} \right) \left(1 \pm \frac{\Delta y}{y} \right)^{-1}.$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in $\Delta y/y$, is $1 \mp (\Delta y/y)$. The relative errors in independent variables are always added. So the error in *z* will be

$$\Delta z = z \left(\frac{\Delta x}{x} + \frac{\Delta y}{y} \right).$$

The above derivation makes the assumption that $\Delta x/x \ll 1$, $\Delta y/y \ll 1$. Therefore, the higher powers of these quantities are neglected.

Q.17 Consider the ratio $r = \frac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a. If the error in the measurement of a is Δa ($\Delta a/a \ll 1$), then what is the error Δr in determining r?

(A)
$$\frac{\Delta a}{(1+a)^2}$$

(B)
$$\frac{2\Delta a}{(1+a)^2}$$

(B)
$$\frac{2\Delta a}{(1+a)^2}$$
 (C) $\frac{2\Delta a}{(1-a^2)}$

(D)
$$\frac{2a\Delta a}{(1-a^2)}$$

Sol: Same As Taught Directly in Theory class of Errors and Measurements Ans (B)

$$r = (1 - a) / (1 + a)$$

:. error in measuring r (error always taken +ve), $\Delta r = \Delta(1-a) + \Delta(1+a) = \Delta a + \Delta a$ r (1-a) (1+a) 1-a 1+a

=
$$\{(1 + a + 1 - a) / 1 - a^2 \}$$
. $\Delta a = 2 \cdot \Delta a / (1 - a^2)$

:.
$$\Delta r = \{2 \cdot \Delta a/(1-a^2)\} \cdot r = 2 \cdot \Delta a/(1-a^2) \cdot \{(1-a)/(1+a)\} = 2\Delta a/(1+a^2)$$

Q.18 In an experiment the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first 1.0 s. For $|x| \ll 1$, $\ln(1+x) = x$ up to first power in x. The error $\Delta \lambda$, in the determination of the decay constant λ , in s⁻¹, is

(D) 0.01

Sol: Same As Taught Directly in Theory class of Errors in Measurements

Ans (C)

 $N = N_0 e^{-\lambda t} = \ln N = \ln N_1 - \lambda t \quad (\ln e = 1)$

differentiating

$$1/N$$
 . $dN = -d_{\lambda}$. t (N , λ are variable for errors)

$$d_{\lambda} = dN / N.t = 40 / (2000 \times 1) = 0.02 \text{ Ans}$$