Jee Advanced 2015 PHYSICS   Paper - 2

INTEGER ANSWER QUESTIONS :-

Q.1. The densities of two solid spheres A and B of same radii R vary with radial distance r as $\rho_A(r)=k(r/R)$ and $\rho_B(r)=k(r/R)^5$ respectively, where k is constant. The moments of inertia of indvidual spheres about axes passing through their centres are I_A and I_B, respectively. If $(I_B/I_A) = n/10$, the valueof *n* is

Q.2. Four harmnic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\pi/3$, $2\pi/3$ and π. When they are superposed, the intensities of resulting wave is nI_0. The value of *n* is

Q.3. For a radioactive material, its activity A and rate of change of its activity R are defined as $A = -(dN/dt)$ and $R = -(dA/dt)$, where N(t) is the number of nuclei at time t. Two radioactive sources P (mean life τ) and Q (mean life 2τ) have same activity at t=0. Their rates of change of activities at $t=2\tau$ are R_P and R_Q ,respectively. If $(R_P/R_Q) = (n/e)$, then the value of *n* is

Q.5. In the following circuit, the current through the resistor R(=2Ω) is I Amperes. The value of *I* is

Q.6. An electron in an excited state of Li^{2+} ion has angular momentum $(3h/2\pi)$. The de Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of *p* is

Q.7. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length l and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass mearer to M is at a distance r = 3I from M, the tension in the rod is zero for m = k(M/288). The value of k is

Q.8. The energy of a system as a function of time t is given as $E(t) = A^2e^{-\alpha t}$, where $\alpha = 0.2\ s^{-1}$. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of E(t) at t=5 s is

MULTIPLE CHOICE QUESTIONS :-

Q.9. A parallel plate capacitor having plates of area S and the plate seperation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio (C_1/C_2) is

(A) 6/5   (B) 5/3   (C) 7/5   (D) 7/3

Q.10. An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown i the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in the relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During the process the piston moves out by a distance *x*. Ignoring the friction between the piston and the cylinder, correct statement(s) is (are)

(A) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $(1/4)P_1V_1$

(B) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

(C) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $(7/3)P_1V_1$

(D) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $(17/6)P_1V_1$

Q.11. A fission reaction is given by $_{92}U^{236} \dashrightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + x + y$, where x and y are two particles. Considering $_{92}U^{236}$ to be at rest, the kinetic energies of the products are denoted by K_{Xe} , K_{Sr} ,K_x (2 MeV) and K_y (2 MeV) , respectively. Let the binding energies per nucleon of $_{92}U^{236}$, $_{54}Xe^{140}$ and $_{38}Sr^{94}$ be 7.5 MeV, 8.5MeV and 8.5 MeV, respectively. Considering different conservation laws, the correct option(s) is (are)

(A) x=n, y=n, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV

(B) x=p, y=e^-, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV

(C) x=p, y=n, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV

(D) x=n, y=n, K_{Sr} = 86MeV, K_{Xe} = 129 MeV

Q.13. In terms of potential difference V , electric current I, permittivity ε_0, permeability μ_0 and speed of light *c*, the dimensionally correct equation(s) is (are)

(A) $\mu_0 I^2 = \varepsilon_0 V^2$ (B) $\varepsilon_0 I = \mu_0 V$ (C) $I = \varepsilon_0 cV$ (D) $\mu_0 cI = \varepsilon_0 V$

Q.14. Consider a uniform spherical charge distribution of radius R_1 centred at the origin *O*. In this distribution, a spherical cavity of radius R_2, centred at *P* with distance OP= a = $R_1 - R_2$ (see figure) is made. If the electric field inside cavity at position **r** is **E**(**r**) then correct statement(s) is (are)

(A) **E** is uniform, its magnitude is independent of R_2 but its direction depends on **r**

(A) **E** is uniform, its magnitude is depends on R_2 and its direction depends on **r**

(A) **E** is uniform, its magnitude is independent of a but its direction depends on **a**

(A) **E** is uniform, both its magnitude and direction depends on **a**

Q.15. In plotting stress versus strain curves for two materials P and Q, a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statement(s) is (are)

(A) P has more tensile strength than Q

(B) P is more ductile than Q

(C) P is more brittle than Q

(D) The Young's modulus of P is more than that of Q

Q.16. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If P(r) is the pressure at r (r < R), then the corrct option(s) is (are)

(A) $P(r = 0) = 0$

(B) $\{P(r = 3R/4)\}/\{P(r = 2R/3)\} = 63/80$

(C) $\{P(r = 3R/5)\}/\{P(r = 2R/5) = 16/21$

(D) $\{P(r = R/2)\}/\{P(r = R/3)\} = 20/27$

PARAGRAPH TYPE QUESTIONS :-

Q.18. Consider two different metallic strips (1 and 2) of the same dimensions (length *l*, width *w*, and thickness *d*) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is same for both the strips the correct option(s) is (are)

(A) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$

(B) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$

(C) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$

(D) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

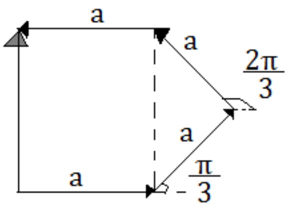
Jee Advanced Solutions Paper 2

1. As taught in numerical class to derive Moment of Inertia of non - uniform sphere.

$$I_A = \int_0^R \frac{2}{3}\left(k\frac{r}{R}.4\pi r^2\, dr\right) r^2 \quad , \quad I_B = \int_0^R \frac{2}{3}\left(k\left(\frac{r}{R}\right)^5 .4\pi r^2\, dr\right) r^2$$

$$\frac{I_A}{I_B} = \frac{9}{15} = \frac{6}{10} = \frac{n}{10} \Rightarrow \boxed{n = 6}$$

2. Taught directly as Q. 56 of S. H. M. in H. C. Verma.



Now,

$I_0 = k\,a^2$ and $I' = k\,(a\sqrt{3})^2$

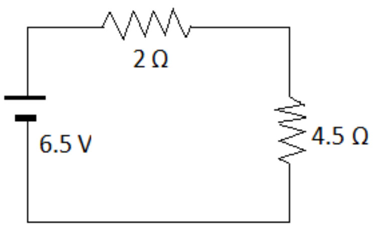
$\therefore\ I' = 3I_0$

3. $A_p = A_0\, e^{-t/\tau}$, $R_p = -d\theta/dt = (A_0/\tau)\, e^{-t/\tau} = A_P/\tau$

$A_Q = A_0\, e^{-t/2\tau}$, $R_Q = -d\theta/dt = (A_0/2\tau)\, e^{-t/2\tau} = A_Q/2\tau$ $\quad$:. $R_P/R_Q = 2/1$

5. Taught directly as Balanced Wheatstone Bridge numerical.

The given circuit will be equivalent to ⇨



So,

$$i = \frac{6.5}{2 + 4.5} = 1\text{ A}$$

6. Taught directly in Theory class of Bohr's Model of H-atom Li^{++}.

For Li^{++} , $l = 3h/2\pi$:. $n = 3$ $\quad$ => $r = (n^2/Z)\, a_0 = 3^2/3\ a_0 = 3a_0$ $\quad$ Also, $mv \cdot 3a_0 = 3h/2\pi$

=> $mv = h/2\pi a_0$ $\quad$ Hence, $\lambda = h/mv = h/(h/2\pi a_0) = 2\pi a_0 = p\pi a_0$ $\quad$ => $p = 3$

7. Taught similar numerical in Gravitation for Equilibrium.

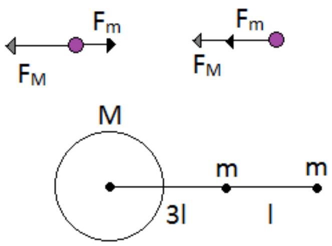
For F = 0

$F_M - F_m = F_M + F_m$

$$\Rightarrow \frac{GMm}{(3l)^2} - \frac{Gmm}{l^2} = \frac{GMm}{(4l)^2} + \frac{Gmm}{l^2}$$

$$\Rightarrow \frac{M}{9} - m = \frac{M}{16} + m$$

$$\Rightarrow \frac{m}{M} = \frac{7}{288} = \frac{k}{288} \Rightarrow \boxed{k = 7}$$



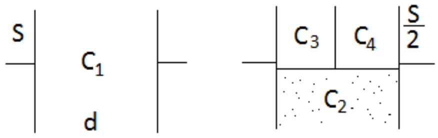
8. Similar to the numerical taught in Errors and measurements.

$E = A^2 \cdot e^{-\alpha t}$

=> $dE/E = 2.\, dA/A + \alpha .t .\, dt/t = 2 \times 1.25 + 0.2 \times 5 \times 1.5 = 2.5 + 1.5 = 4\ \%$ $\quad$:. Ans (a , c)

9. Taught directly in Q. 56 (a),(c) of capacitors in H.C.V.

$C_1 = \frac{\varepsilon_0 S}{d}$ $\quad C_2 = \frac{2\varepsilon_0 S/2}{d} = \frac{\varepsilon_0 S}{d} = C_1$



$C_3 = \frac{2\varepsilon_0 S/2}{d/2} = \frac{2\varepsilon_0 S}{d} = 2C_1$ $\quad C_4 = \frac{4\varepsilon_0 S/2}{d/2} = \frac{4\varepsilon_0 S}{d} = 4C_1$

$C_{eq} = \frac{2C_1 \cdot 4C_1}{2C_1 + 4C_1} = \frac{4}{3} C_1$ $\quad$ $C_{eq} = C_1 + \frac{4}{3} C_1 = \frac{7}{3} C_1$ $\quad$ Ans. D

10. Similar to solved example of Heat & Thermodynamics H.C.V.

Hence, Ans. A, B, C.

12. Similar to that taught in Fluid Mechanics and Terminal Velocity.

For equilibrium ,

$B_P + B_Q = W_P + W_Q$

$\sigma_1 + \sigma_2 = \rho_1 + \rho_2$ ----------- (1)

Terminal velocity, $v_P = \frac{2}{9n_2} r^2 g\,(\rho_1 - \sigma_2)$

$v_Q = \frac{2}{9n_1} r^2 g\,(\rho_2 - \sigma_1)$

$\frac{v_P}{v_Q} = \frac{n_1(\rho_1 - \sigma_2)}{n_2(\rho_2 - \sigma_1)} = -\frac{n_1}{n_2}$

(A) and $\vec{v}_P \cdot \vec{v}_Q < 0$

If v_P is up, v_Q will be down and vice - versa

13. Directly taught in Units and Dimensions of given terms.

$[V] = [V/q] = [(M L^2 T^{-2}) / I T] = [M L^2 T^{-3} I^{-1}]$

$[\varepsilon_0] = [q_1 q_2 / 4\pi r^2 F] = [IT . IT / L^2 MLT^{-2}] = [M^{-1} L^{-3} T^4 I^2]$

$[\mu_0] = [1/\varepsilon_0 c^2] = [1 / M^{-1} L^{-3} T^4 I^2 . L^2 T^{-2}] = [M L T^{-2} I^{-2}]$ $\quad$:. Ans. A , C

14. Directly taught Numerical in Theory class of ELectrostatics.

Field is uniform (same) for all points inside the cavity and equal to $\mathbf{E} = -\rho\mathbf{a}/3\varepsilon_0$. :. Ans. D

15. Directly taught in theory class of Stress Strain curve for Ductile materials.

Y = stress/strain :. $Y_Q > Y_P$ (Larger slope in Q)

Graph of P has more length. Hence, P has more tensile strength than Q .

Also, Q starts bending first so P is ductile than Q .

17. Directly taught as Q. 29 Hall Effect of H.C.V.

$j = V_d = \frac{j}{ne} = \frac{I}{neWd}$, $F_m = e V_d . B = eB . \frac{I}{ne\,Wd}$

Steady state,

$eE = F_m$ $\quad eE = \frac{B\,I}{n\,Wd}$ $\quad E = \frac{B\,I}{ne\,Wd}$

p . d. , $V = E.\,W = \frac{B\,I}{ne\,Wd} . W = \frac{B\,I}{ne\,d}$

Ans. A , C

$d_1 = 2d_2$, $V_1 = V_2 / 2$ $\quad \therefore \quad V_2 = 2V_1$

18. If $V = I\,B/ ned$, if $n_1 = 2n_2$, $v_1 = v_2 / 2$, $v_2 = 2v_1$

and , $n_1 = n_2$, $B_1 = 2B_2$, $v_1 = 2v_2$ $\therefore$ Ans. A , C

19. Directly taught in theory class of Optical Fibres.